**Operations Analytics: Advanced Problems**

DEA Analysis, Transportation and Job Assignment Problems

BEM3062: Operations Analytics

Dr. Stuart So

06/01/2025

Word Count: 2850

**Project 1: Data Envelopment Analysis** (**DEA) Analysis**

The board of 4 elementary schools has implemented a series of standards of learning (SOL) tests in reading, math, and history, and has also taken 3 additional relevant measures. This scenario is a DEA problem as this methodology was invented to manage services similar to what a school provides and measures the efficiency of decision-making units (DMUs), which in this case are Alton, Beeks, Carey and Delancey elementary schools. This can be formulated as a mathematical model and solved through simple linear programming (LP) because it matches with its criteria.

* A linear equation can be constructed from the objective (‘Superprof’,2019). For example, Delancey’s maximum relative efficiency (MRE) can be found through identifying the decision variables and implementing an objective function.

**Objective Function**

Considering the 6 measures which were stated above, they can be divided into the inputs and outputs of the models. The inputs are as follows.

* Teacher-to-student ratio: This would the total number of teachers divided by the total number of students and can represent each school’s ability to give each student personalized attention.
* Supplementary funds/student: This represents the school’s ability to provide students with improved resources such as smartboards, additional materials like textbooks, the technology to online learning like meetings and resource dashboards, better learning environments and even new buildings and classrooms.
* Average educational level of parents: This represents the calibre of background that the student comes from which is essentially their platform for learning.

The decision variables linked to inputs are the weights attributed to each input and are X1,X2,X3 for each respective measure.

The outputs of the model are the average SOL scores for the following.

* Reading
* Math
* History

The decision variables linked to outputs are the weights attributed to each output and are X4, X5 ,X6 for each respective measure.

These are considered by the board as the inputs and outputs as the relative efficiency of each school is measured on how the most critical factors that can potentially affect the SOL scores of young kids, and how well the school can convert their resources and manage these factors to produce the best SOL scores possible.

The following objective function to find Delancey’s MRE.

Maximize Z = 81X4 + 73X5 + 69X6

These values are used because the board ultimately considers the outputs SOLs for Delancey.

* This scenario is linear problem because the constraints can be written as linear inequalities or equations (‘Superprof’,2019).

Delancey’s objective is subject to,

0.06X1 + 260X2 + 11.3X3 >= 86X4 + 75X5 + 71X6

0.05X1 + 320X2 + 10.5X3 >= 82X4 + 72X5 + 67X6

0.08X1 + 340X2 + 12X3 >= 81X4 + 79X5 + 80X6

0.06X1 + 460X2 + 13.1X3 >= 81X4 + 73X5 + 69X6

This states that the value of each schools’ inputs is greater or equal than its outputs.

This is because it is impossible for school’s efficiency to be greater than 100%. Therefore, a schools’ MRE is less than or equal to 1. Efficiency is equal to sum of outputs divided by the sum of inputs. Therefore, this generates the following inequality.

Value of a school’s outputs

1 >=

Value of a school’s inputs

The above constraints are essentially a re-arrangement of this inequality for each school.

Constraints on the weights.

0.06X1 + 460X2 + 13.1X3 = 1

The weights of a regression model in mathematics generally sum to 1. Additionally, this forces the efficiency score to be less than 1, which is ideal.

All values are non-negative.

* Finally, in order for this problem to be solved through LP, the resources involved must be finite and the objective must be quantifiable (Basumallick,2022), which is correct in this scenario.

This function would yield the following results through spreadsheet modelling.

**Spreadsheet Modelling**

A screenshot of a spreadsheet

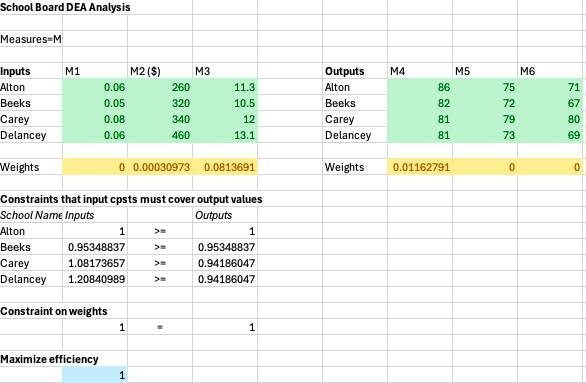
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A screenshot of a computer

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This indicates that Delancey’s MRE is 0.8582.

This is repeated for the schools.

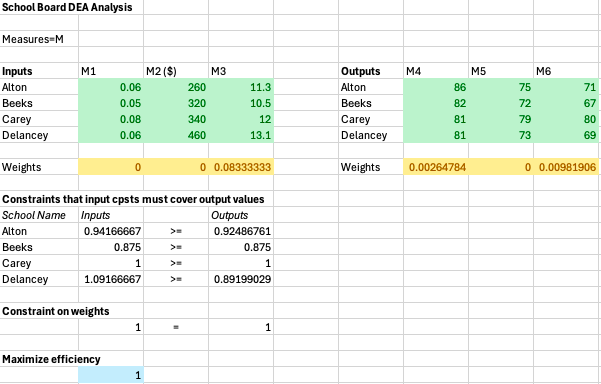


Alton’s MRE is 1.

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Beeks’ MRE is 1



Finally, Carey’s MRE is 1

**Conclusion**

It can be concluded that although an MRE of approximately 0.8 seems high at first, when compared to the other schools, Delancey is relatively inefficient. Considering the efficiency formula given above, this means that the school isn’t utilizing its inputs as effectively as the other school (approximately 20% less effectively).

To evaluate, this result doesn’t mean that Delancey isn’t performing well as it is relative.

Additionally, an efficiency score of 1 for the other schools doesn’t mean that they’re perfectly run for the same reason. It is also important to remember that managing a set of schools is harder than owning just 1 school, for example. A board has a set of resources, and they must allocate them effectively and it is extremely difficult to be perfect in this regard.

**Recommendations for Delancey**

* The average education level of parents is the highest and therefore, many of them are college graduates. This might indicate that these parents are encouraging the following.
* Children to focus on STEM related subjects by acting as role models in this regard even when the children are young (Gülhan,2023). This is alluded to by the school’s below average performance in History.
* Research suggests that children from wealthier backgrounds tend to take up more extracurricular activities such as music lessons (Social Mobility Commission,2019). This might shift the focus of younger kids slightly away from their studies and alludes to Delancey’s below average SOLs.

Delancey must invest its well-above average available funds in creating an enivronment that encourages an interest in history and learning in general and even alter the syllabus so it highlights the importance of learning history. They can also invest these funds in AI-powered virtual classrooms that better tailor to a student’s needs as it analyzes each student’s characteristics, which is especially useful in special needs education, and can make recommendations for curriculums, which optimizes an education system (fintelics,2023).

**Recommendations for the board**

A comparison between a board’s own schools is critical to effectively manage a set of resources. The next step would be to conduct a similar project but externally, with well-established school boards that own a set of schools or with schools around the areas of Alton, Beeks, Carey and Delancey. This might give the board an idea of where to invest and grow, which potentially attract more students’ applicants and more skilled employees who could increase the efficiency of a school’s services.

**Project 2: Transportation Problem**

Exeter Transport Company’s scenario is a minimum cost network flow problem. There are many methods to solve it, and each would yield different results. It is important to test each method and compare the results to find the optimal solution. The various methodologies can be seen below.

**Supply-Demand Network Diagram**

The following visualizes Exeter Transport Company’s shipping scheme.

**A diagram of a graph

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This visualizing technique displays their plan simply and clearly. Here, the ovals represent the nodes (geographical locations), and the arrows represent the nodes (shipping route). Additionally, the upper capacity limits have demand and supply of the mills and silos respectively are given on either side. To develop a model, this diagram can be built on through a tableau.

**Transportation Tableau**

Below is the corresponding tableau generated.

**A grid of squares with numbers

Description automatically generated**

To simplify, the names of silos and mills have now been ignored but will be highlighted later. Each box here represents a unit transportation cost, and the corresponding supply and demand related to that cost are on either axis of the matrix.

This tableau is crucial as allows for computation of basic feasible solutions (BFS).

The feasible solutions that will be computed below are a set of non-negative values that fulfill the constraints, and the optimal solution illustrates the minimum total transportation cost.

Here demand and supply are considered as the constraints of the model, and this illustrates that this is a balanced transportation problem as total demand equals total supply.

**Identifying BFS**

The following are 3 methods of doing so.

*Northwest Corner Method*

The following is a handwritten computation of the northwest corner method.



As seen above, this method a developed from the transportation tableau. As this is balanced problem, no dummy rows or columns were necessary. The unit cost of 10 is in the northwest corner of matrix and therefore, that is the first allocation of the process. The corresponding demand limit is 5. Due to this, the appropriate supply capacity is only reduced to 10 from 15. A capacity of 5 is then allocated to this unit cost. Once the supply or demand of respective rows or columns have been limited to 0, it is crossed out. The procedure is continued until full cost allocation and the subsequent allocation is a unit cost of 2 with a capacity of 10, 7 with 5, 20 with 5, 9 with 15 and finally 18 with an allocation of 10 is left.

This produces the following result.10(5) + 2(10) + 7(5) + 9(15) + 20(5) + 18(10) = 610. Therefore, the total transportation cost is $61000.

This is the simplest method and produces a BFS the quickest out of the next few procedures.

Below is the least cost method, which is widely considered as an optimal method for finding a BFS.

*Least-Cost Method*

The figure below is a handwritten computation of this method.

*A graph with numbers and lines

Description automatically generated with medium confidence*

There were a few important steps in this process. The minimum costs were allocated in this order: 2 with a capacity of 15 allocated to it and the corresponding row and column were crossed out as they both had a value of 15. Subsequently, 4 with a capacity of 5, 9 with 15, 18 with 5 and finally 20 with a capacity remained.

This produces the following result. 2(15) + 4(5) + 9(15) + 18(5) + 20(10) = 475. Therefore, the total transportation cost is $47500.

Although the Northwest method produces an initial BFS easily, this method produces a lower cost solution.

*Vogel’s Method*

It achieved through the following handwritten computation.

*A close-up of a paper

Description automatically generated*

This process shares similarities with the minimum cost method. Here, row and column penalties are computed and the minimum cost of row or column with the lowest penalty is identified, and similar procedure is followed as the previous method. Here, firstly a cost 2 is pinpointed out and a capacity of 15 is allocated to it. Then 4 with 5, 9 with 15, 18 with 5 and finally 20 with 10.

This produces the following result 4(5) + 2(15) + 9(15) + 18(5) + 20(10) = 475. Therefore, the total transportation cost is $47500.

This procedure for finding the BFS tries to avoid extremely high shipping costs.

For a more comprehensive solution to the problem, a mathematical model of the Exeter Transport Company scenario would be the best approach.

**Mathematical Model**

This problem can be formulated as linear programming model with the objective of minimizing the total transportation cost because it follows a similar structure to the previous project. A linear equation can be constructed from the minimization objective with decision variables, an objective function, constraints which can be written as linear inequalities or equations (‘Superprof’,2019). This is shown below.

*Decision Variables*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **To** |  |  |  |  |
| **From** |  | Chicago | St. Louis | Cincinnati | Detroit |
|  | Kansas City | X1 | X4 | X7 | X10 |
|  | Omaha | X2 | X5 | X8 | X11 |
|  | Des Moines | X3 | X6 | X9 | X12 |

This can be used to produce the following objective function.

*Objective Function*

Minimize C = 10X1 + 12X2 + 4X3 + 2X4 + 7X5 + 14X6 + 20X7 + 9X8 + 16X9 + 11X10 + 20X11 + 18X12

Subject to,

Demand Constraints:

X1 + X2 + X3 >= 5

X4 + X5 + X6 >= 15

X7+ X8 + X9 >= 15

X10 + X11 + X12 >= 15

Supply Constraints:

X1 + X4 + X7 + X10 <= 15

X2+ X5+ X8 + X11 <= 25

X3+ X6 + X9 + X12 <= 10

Using spreadsheeting modelling, the following has been computed.

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This has produced 435 as its result. Therefore, the total transportation cost here is $43500.

**Conclusion**

The LP model has produced the result with the lowest total cost. The least-cost and Vogel’s method produced the same result as they essentially followed the same allocation. Despite the convenience of the northwest corner method, its total cost was significantly higher. Therefore, the mathematical method is optimal for this problem.

**Recommendations**

* The $4000 difference between the heuristic approaches (least-cost and Vogel’s) isn’t significant and should therefore be used when a quick and easy result is needed.
* Although, the minimization of transportation cost is a crucial goal for the Exeter Transport Company, measures taken to be more sustainable, ensure delivery time goals are met and balancing customer satisfaction are other critical goals which mustn’t be ignored, especially with increased need for businesses to be more sustainable (Rashid & Islam,2023).
* It is also important to not disregard other processes such as northwest corner method because transportation costs tend to fluctuate and can often be volatile. Therefore, having many angles of looking at an issue is always useful. It can therefore be recommended that although an LP approach should be prioritized, resources should also be spent in finding a BFS using each of the above methods (Rashid & Islam,2023).

**Project 3: Job Assignment**

There are 2 prime methods of solving this method and when the results of each are compared an optimal result can be identified. These are the Hungarian Algorithm and through a mathematical model. Both are used below.

**The Hungarian Algorithm**

Out of all the possible graphical methods, this is the best for solving this problem as it is superior for when each activity is done by a different person and the objective is to find the minimum cost to complete all the activities. It follows the below steps.

* Construction of matrix of the bids by each person for each job.
* Subtracting the minimum value from each row.
* Subtracting the minimum value from each column from the new set of values.

A sheet of paper with numbers and letters

Description automatically generated

The first 3 steps are done here, and a new set of values have been found.

* Cover all the zeros with the minimum amount of horizontal and vertical lines.

**A close-up of a paper

Description automatically generated**

The number of lines used was 3, which is the equal to the dimensions of the matrix (3 by 3). This indicates that the problem is balanced, and no alterations are necessary. Therefore, it is possible to move onto the next steps.

* Look at which row has the least number of zeros and assign that point in the matrix and cross out the corresponding column. Follow the same process until each job is assigned

A graph with writing on it

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This is successfully done here. As seen above, the idea here is that once, for example, Terri has been assigned to wash, that column (job) is now finished. Therefore, the only other possible job for John is paint. This is ideal as the row ‘John’ also has a zero under ‘Paint’.

The following are the job assignments through the Hungarian Method.

* John to paint
* Karen to mow
* Terri to wash

If this occurs, the total cost would be 9 + 10 + 8 = $27.

It is now appropriate to construct a mathematical model around this problem and it is achieved through spreadsheet modelling.

**Mathematical Model**

This problem is a special kind of transportation problem (possibility of transforming the values into a matrix). More specifically, it is a special kind of least-cost problem as it is a minimization objective. Therefore, like project 2, a simple LP model can be generated to find a solution (‘Superprof’,2019). The solution is found through the following steps.

* Objective function

With the objective of minimizing total cost, the following is the objective function of the model with the decision variables being the weights of each potential cost to Klyne (Xi).

Minimize C= 15X1 + 9X2 + 10X3 + 10X4 + 15X5 + 12X6 + 9X7 + 10X8 + 8X9

Subject to, constraints on how many jobs each child can do and how many children are allowed to take on each job,

X1 + X2 + X3 = 1

X4 + X5 + X6 = 1

X7 + X8 + X9 =1

X1 + X4 + X7 = 1

X2 + X5 + X8 = 1

X3 + X6 + X9 =1

* Spreadsheet modelling

The following result was achieved through this method.

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It is found that the total cost is $27.

**Conclusion**

Both the Hungarian and the LP model have produced the same result that assigning Karen to mow, John to paint and Terri to wash and minimize the total cost to $27. It can be said the Hungarian is ideal for this kind of assignment problem because it usually finds an optimal solution for simple issues like Klyne’s and is faster than complex LP models (‘Stack Exchange’,2022). However, if a situation was to become larger and have many more constraints, a LP model would be more accurate. Spreadsheet models are also more flexible as it is a program and therefore, it is only necessary to input new values to change the outcome completely. Additionally, it is not too difficult to add and alter constraints or the program (‘Stack Exchange’,2022).

With this information, it is recommended that both approaches should be used they have many differences and therefore complement each other in tackling any assignment problems in the future.

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